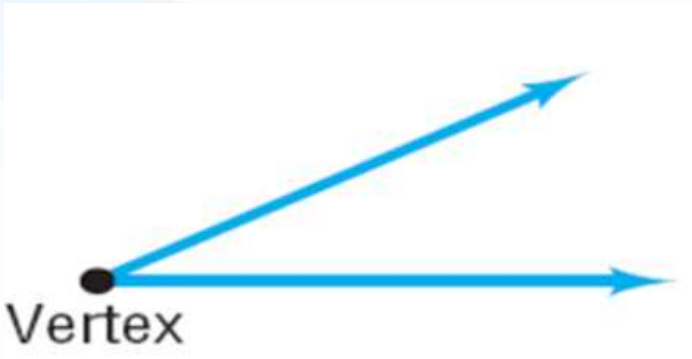
Trigonometric Functions

Basic Terminology

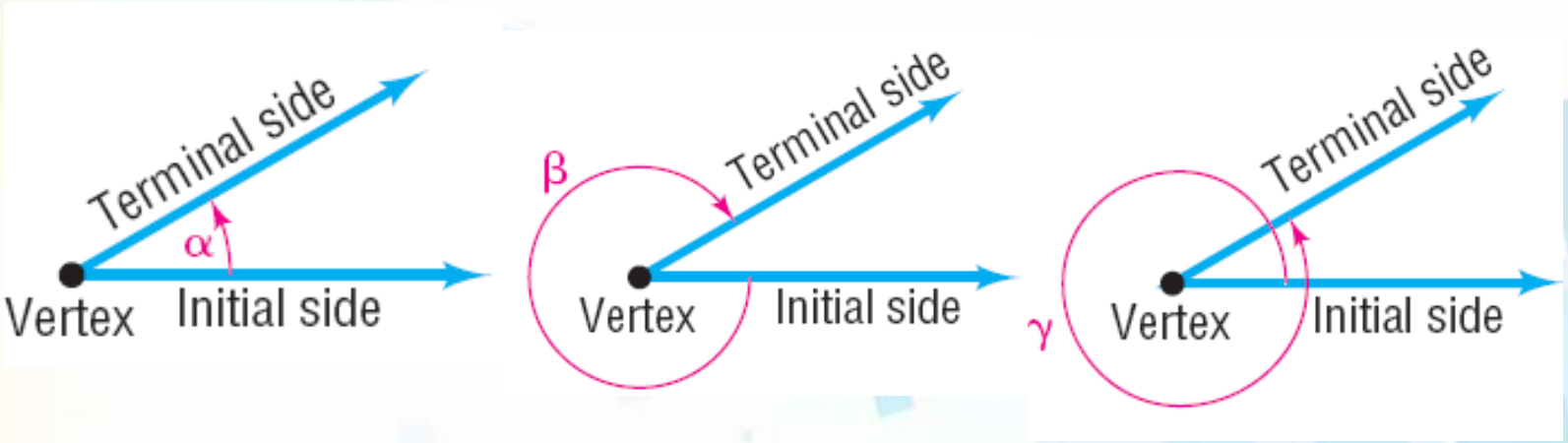
- Ray: A half-line starting at a $vertex\ V$

- $Angle$: Two rays with a common vertex



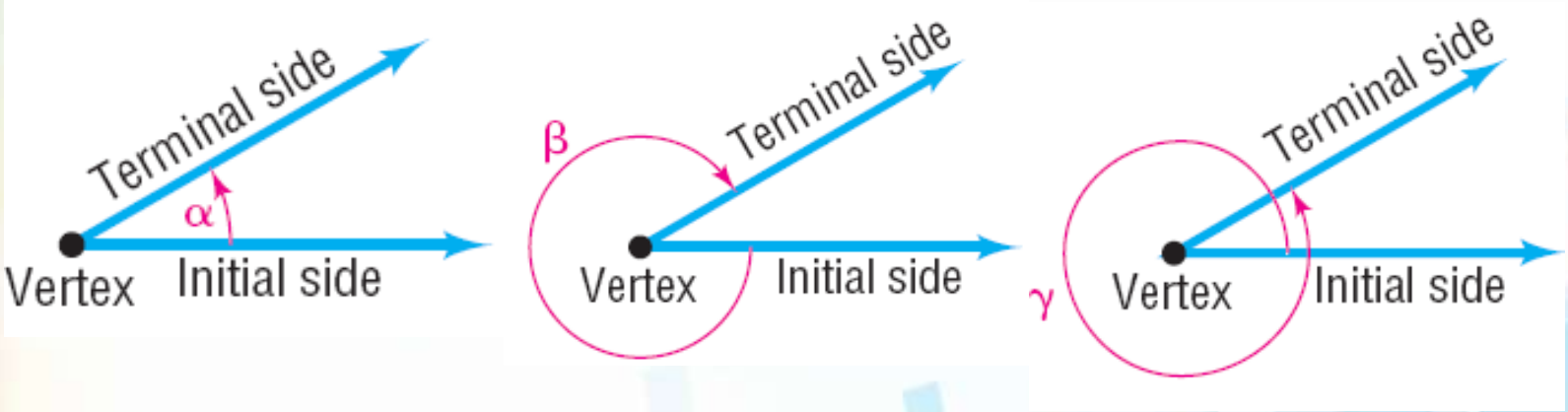
Basic Terminology

- *Initial side* and *terminal side*: The rays in an angle
  - Angle shows direction and amount of rotation
  - Lower-case Greek letters denote angles



Basic Terminology

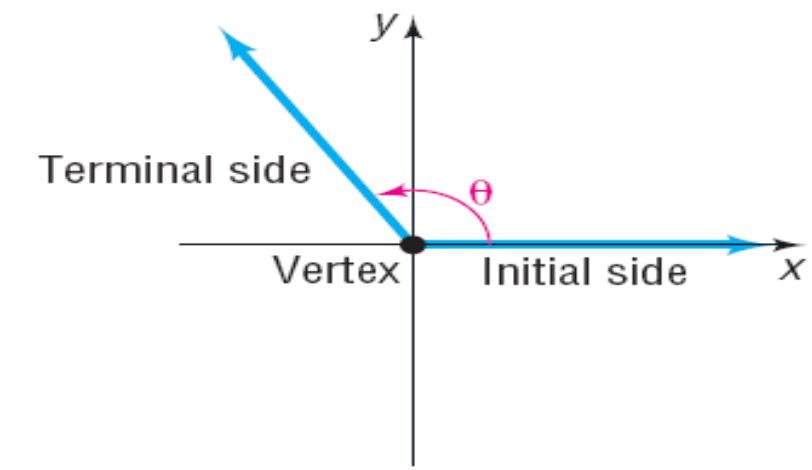
- *Positive angle*: Counterclockwise rotation
- *Negative angle*: Clockwise rotation
- *Coterminal angles*: Share initial and terminal sides

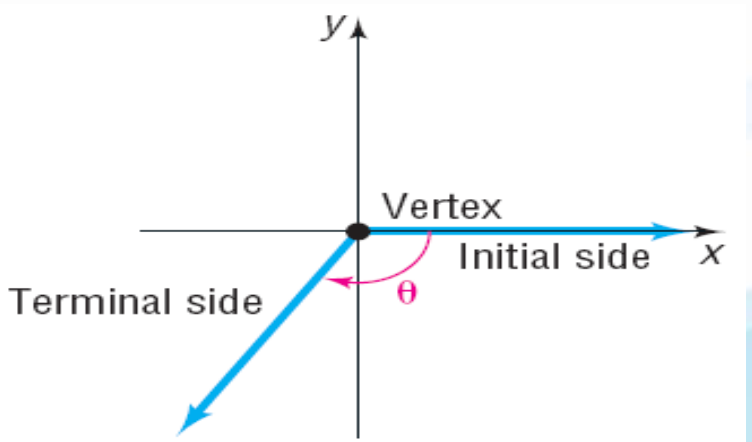


Positive angle Negative angle Positive angle

Basic Terminology

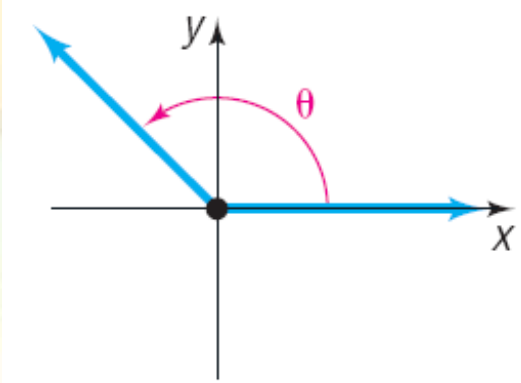
- *Standard position*:
  - Vertex at origin
  - Initial side is positive x-axis



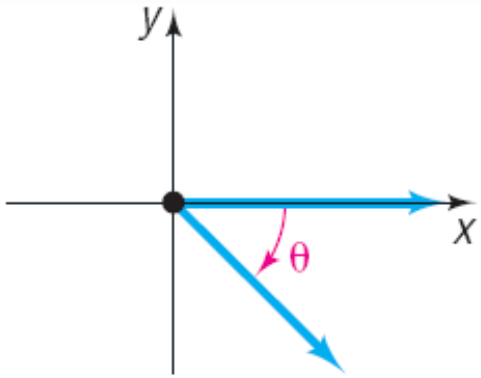


Basic Terminology

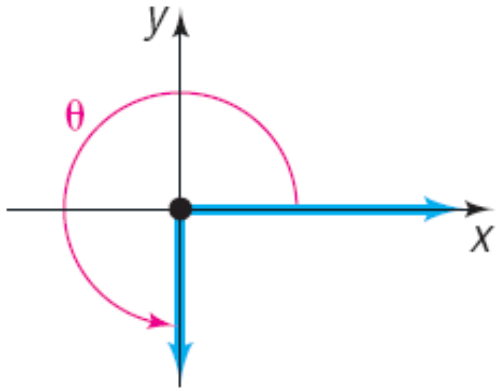
- *Quadrant angle*: Angle in standard position that doesn't lie in any quadrant



Lies in quadrant II



Lies in quadrant IV



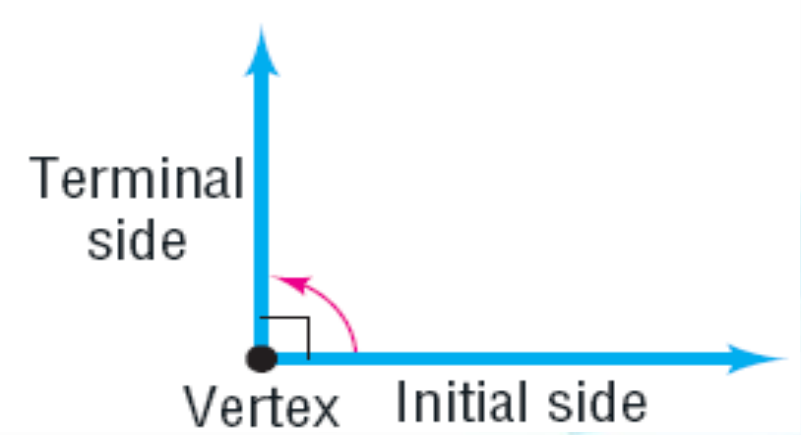
Quadrantal angle

Measuring Angles

- Two usual ways of measuring
  - $Degrees$
    - 360° in one rotation
  - $Radians$
    - 2π radians in one rotation

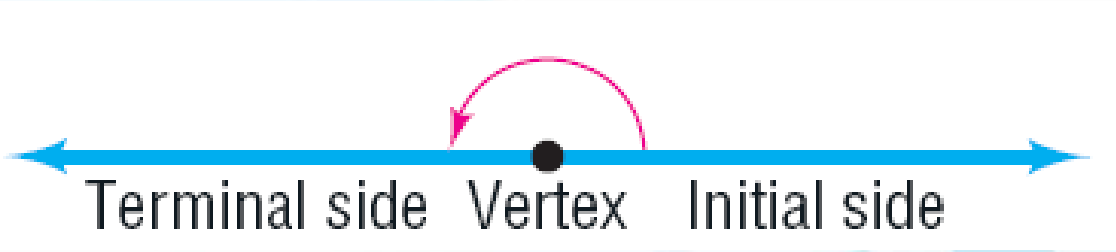
Measuring Angles

- *Right angle*: A quarter revolution
  - A right angle contains
    - $\frac{1}{4}(360)^\circ$
    - $\frac{\pi}{2}$ radians



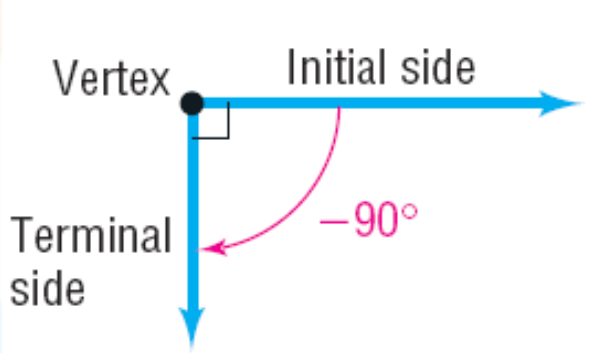
Measuring Angles

- *Straight angle*: A half revolution.
  - A straight angle contains:
    - 180°
    - π radians

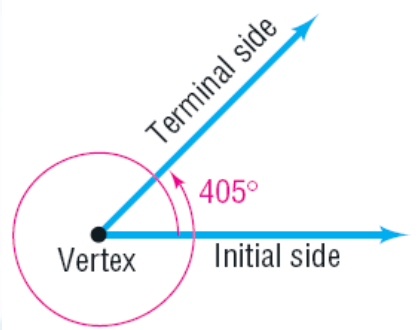


Measuring Angles

- Negative angles have negative measure



- Multiple revolutions are allowed



Degrees, Minutes and Seconds

- One complete revolution = 360°
- One $minute$:
  - One-sixtieth of a degree
  - One minute is denoted $1'$
  - $1^\circ = 60'$
- One $second$:
  - One-sixtieth of a minute
  - One second is denoted $1''$
  - $1' = 60''$

Radians vs. Degrees

- Example. Convert each angle in degrees to radians and each angle in radians to degrees

(a) Problem: 45°

Answer:

(b) Problem: {270°

Answer:

(c) Problem: 2 radians

Answer:

Radians vs. Degrees

- Measurements of common angles

Degrees	0°	30°	45°	60°	90°	120°	135°	150°	180°
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
Degrees		210°	225°	240°	270°	300°	315°	330°	360°
Radians		$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π

Exact Values for Quadrantal Angles

- $Quadrantal\ angles$ correspond to integer multiples of 90° or of $\frac{\pi}{2}$ radians

Exact Values for Quadrantal Angles

- Example. Find the values of the trigonometric functions of θ

  Problem: θ = = 90° $\frac{\pi}{2}$

  Answer:

Exact Values for Quadrantal Angles

- Example. Find the values of the trigonometric functions of θ

  Problem: $\theta = \pi = 180^\circ$

  Answer:

Exact Values for Quadrantal Angles

- Example. Find the values of the trigonometric functions of θ

  Problem: $\theta =$ $= 270^\circ$ $\frac{3\pi}{2}$

  Answer:

Exact Values for Quadrantal Angles

Quadrantal Angles							
θ (Radians)	θ (Degrees)	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
0	0°	0	1	0	Not defined	1	Not defined
$\frac{\pi}{2}$	90°	1	0	Not defined	1	Not defined	0
π	180°	0	−1	0	Not defined	−1	Not defined
$\frac{3\pi}{2}$	270°	−1	0	Not defined	−1	Not defined	0

Exact Values for Quadrantal Angles

- Example. Find the exact values of

(a) Problem: sin ({90°)

Answer:

(b) Problem: cos (5π)

Answer:

Exact Values for Standard Angles

- Example. Find the values of the trigonometric functions of θ

  Problem: $\theta =$ $= 45^\circ$ $\frac{\pi}{4}$

  Answer:

Exact Values for Standard Angles

- Example. Find the values of the trigonometric functions of θ

  Problem: $\theta =$ $= 60^\circ \frac{\pi}{3}$

  Answer:

Exact Values for Standard Angles

- Example. Find the values of the trigonometric functions of θ

  Problem: $\theta =$ $= 30^\circ$ $\frac{\pi}{6}$

  Answer:

Exact Values for Standard Angles

θ (Radians)	θ (Degrees)	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$\frac{\pi}{6}$	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{4}$	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$\frac{\pi}{3}$	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

Exact Values for Standard Angles

- Example. Find the values of the following expressions

(a) Problem: $\sin(315^\circ)$

Answer:

(b) Problem: $\cos(\{120^\circ)$

Answer:

(c) Problem: $\tan \frac{5\pi}{6}$

Answer:

Approximating Values Using a Calculator

- Example. Use a calculator to find the approximate values of the following. Express your answers rounded to two decimal places.

  (a) Problem: sin 57°

  Answer:

  (b) Problem: cot {153°

  Answer:

  (c) Problem: sec 2

  Answer:

Periods of Trigonometric Functions

- Periodic Properties:

$$\sin(\theta + 2\pi) = \sin\theta$$
$$\cos(\theta + 2\pi) = \cos\theta$$
$$\tan(\theta + \pi) = \tan\theta$$
$$\csc(\theta + 2\pi) = \csc\theta$$
$$\sec(\theta + 2\pi) = \sec\theta$$
$$\cot(\theta + \pi) = \cot\theta$$

- Sine, cosine, cosecant and secant have period 2π
- Tangent and cotangent have period π

Periods of Trigonometric Functions

- Example. Find the exact values of

(a) Problem: $\sin(7\pi)$

Answer:

(b) Problem: $\cos \dfrac{37\pi}{6}$

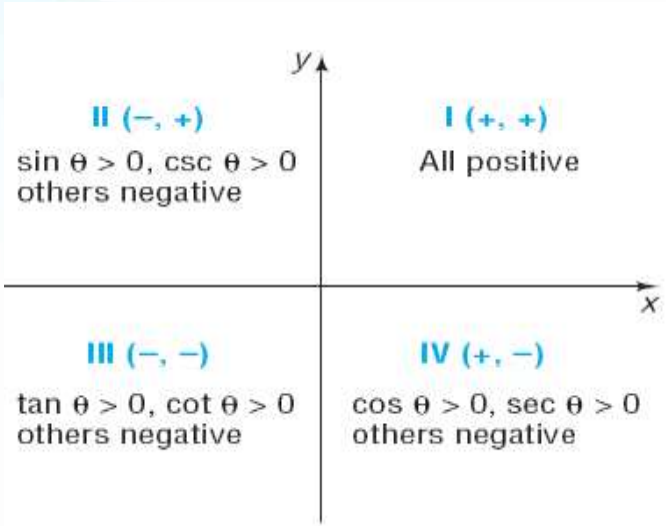
Answer:

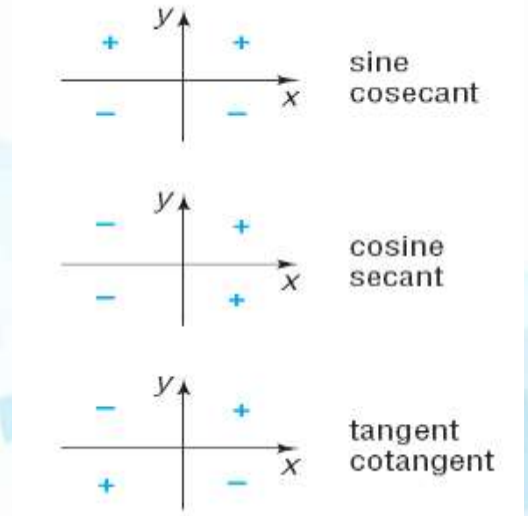
(c) Problem: $\tan \dfrac{19\pi}{4}$

Answer:

Signs of the Trigonometric Functions

Quadrant of θ	sin θ, csc θ	cos θ, sec θ	tan θ, cot θ
I	Positive	Positive	Positive
II	Positive	Negative	Negative
III	Negative	Negative	Positive
IV	Negative	Positive	Negative





Quotient Identities

- Example.

  Problem: Given $\sin\theta = \frac{\sqrt{47}}{10}$ and $\cos\theta = \frac{\sqrt{53}}{10}$ find the exact values of the four remaining trigonometric functions of θ using identities.

  Answer:

Pythagorean Identities

- Example. Find the exact values of each expression. Do not use a calculator

(a) Problem: $\cos 20^\circ \sec 20^\circ$

Answer:

(b) Problem: $\tan^2 25^\circ \{ \sec^2 25^\circ$

Answer:

Pythagorean Identities

- Example.

  Problem: Given that $\tan\theta = -\frac{8}{9}$ and that θ is in Quadrant II, find cos θ.

  Answer:

Even-Odd Properties

- Theorem. [Even-Odd Properties]

$$\sin(\{\theta) = \{\sin(\theta)$$
$$\cos(\{\theta) = \cos(\theta)$$
$$\tan(\{\theta) = \{\tan(\theta)$$
$$\csc(\{\theta) = \{\csc(\theta)$$
$$\sec(\{\theta) = \sec(\theta)$$
$$\cot(\{\theta) = \{\cot(\theta)$$

- Cosine and secant are even functions
- The other functions are odd functions

Even-Odd Properties

- Example. Find the exact values of

(a) Problem: sin({30°)

Answer:

(b) Problem: $\cos\left(-\frac{\pi}{4}\right)$

Answer:

(c) Problem: $\cot\left(-\frac{33\pi}{4}\right)$

Answer:

Fundamental Trigonometric Identities

- Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

- Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

- Pythagorean Identities

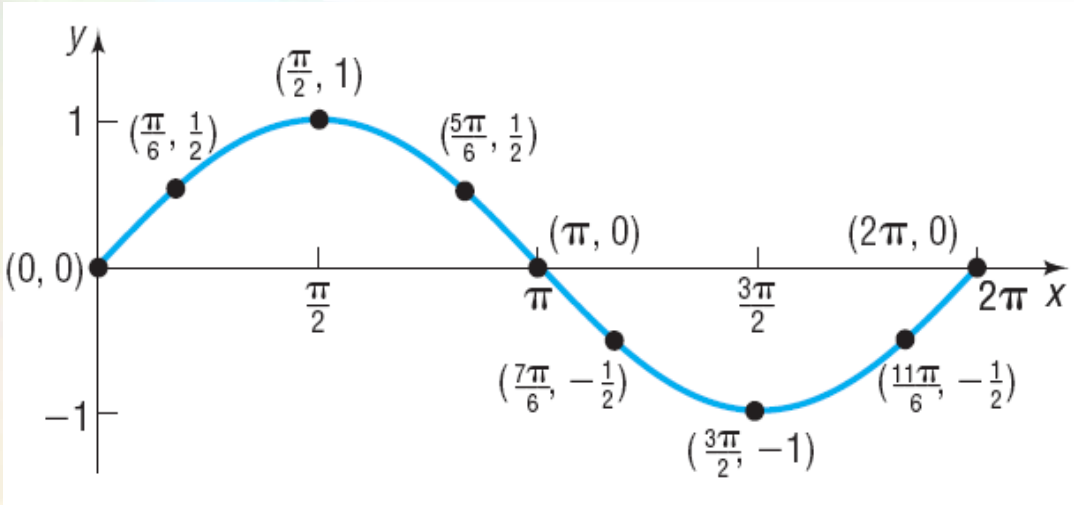
$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

- Even-Odd Identities

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta \qquad \tan(-\theta) = -\tan\theta$$
$$\csc(-\theta) = -\csc\theta \qquad \sec(-\theta) = \sec\theta \qquad \cot(-\theta) = -\cot\theta$$

Graphing the Sine Function

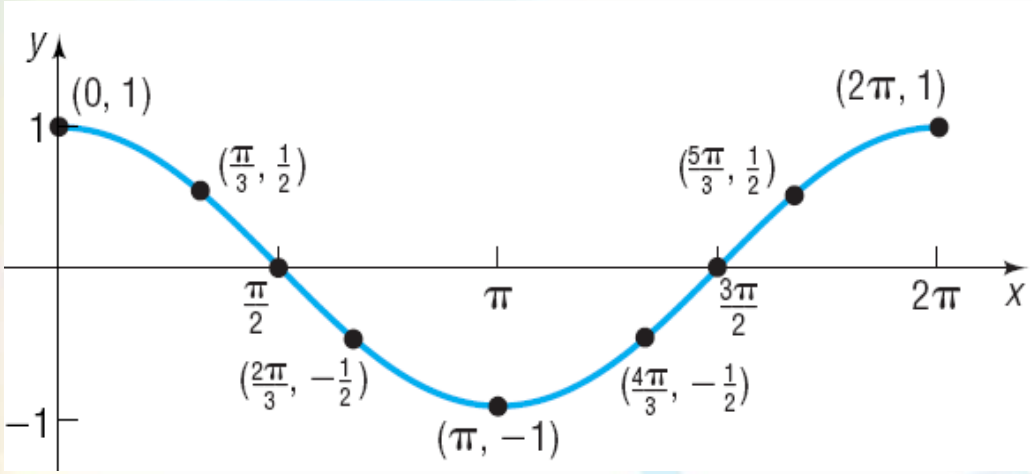
- **Periodicity**: Only need to graph on interval $[0, 2\pi]$ (One *cycle*)
- Plot points and graph



x	y = sin x	(x, y)
0	0	(0, 0)
$\frac{\pi}{6}$	$\frac{1}{2}$	$\left(\frac{\pi}{6}, \frac{1}{2}\right)$
$\frac{\pi}{2}$	1	$\left(\frac{\pi}{2}, 1\right)$
$\frac{5\pi}{6}$	$\frac{1}{2}$	$\left(\frac{5\pi}{6}, \frac{1}{2}\right)$
π	0	$(\pi, 0)$
$\frac{7\pi}{6}$	$-\frac{1}{2}$	$\left(\frac{7\pi}{6}, -\frac{1}{2}\right)$
$\frac{3\pi}{2}$	−1	$\left(\frac{3\pi}{2}, -1\right)$
$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\left(\frac{11\pi}{6}, -\frac{1}{2}\right)$
2π	0	$(2\pi, 0)$

Graphing the Cosine Function

- **Periodicity:** Again, only need to graph on interval [0, 2π] (One $cycle$)
- Plot points and graph1

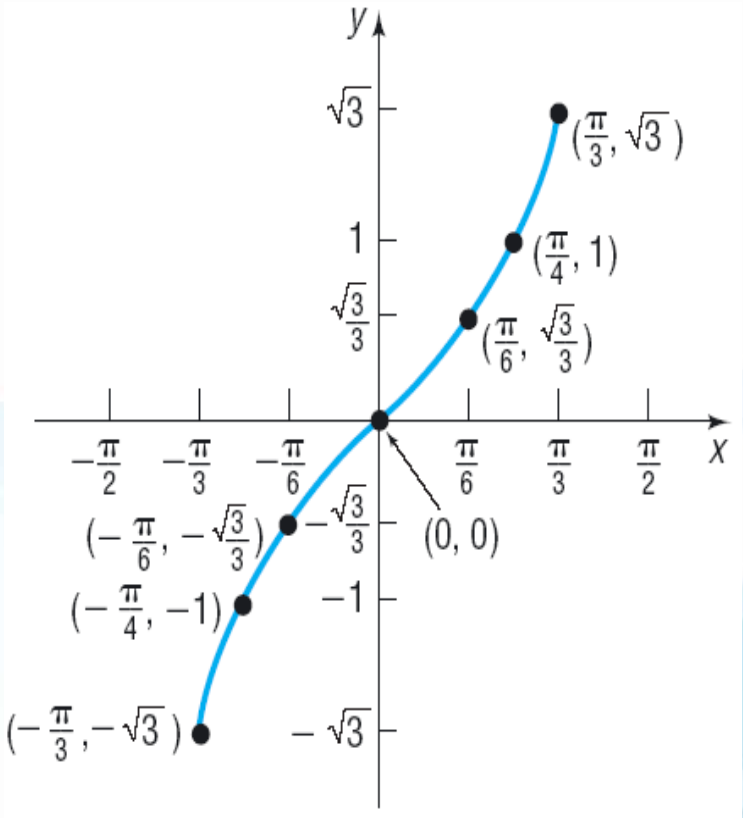


x	y = cos x	(x, y)
0	1	(0, 1)
$\frac{\pi}{3}$	$\frac{1}{2}$	$\left(\frac{\pi}{3}, \frac{1}{2}\right)$
$\frac{\pi}{2}$	0	$\left(\frac{\pi}{2}, 0\right)$
$\frac{2\pi}{3}$	$-\frac{1}{2}$	$\left(\frac{2\pi}{3}, -\frac{1}{2}\right)$
π	−1	$(\pi, -1)$
$\frac{4\pi}{3}$	$-\frac{1}{2}$	$\left(\frac{4\pi}{3}, -\frac{1}{2}\right)$
$\frac{3\pi}{2}$	0	$\left(\frac{3\pi}{2}, 0\right)$
$\frac{5\pi}{3}$	$\frac{1}{2}$	$\left(\frac{5\pi}{3}, \frac{1}{2}\right)$
2π	1	$(2\pi, 1)$

Graphing the Tangent Function

- Periodicity: Only need to graph on interval $[0, \pi]$
- Plot points and graph

x	$y = \tan x$	(x, y)
$-\frac{\pi}{3}$	$-\sqrt{3} \approx -1.73$	$\left(-\frac{\pi}{3}, -\sqrt{3}\right)$
$-\frac{\pi}{4}$	-1	$\left(-\frac{\pi}{4}, -1\right)$
$-\frac{\pi}{6}$	$-\frac{\sqrt{3}}{3} \approx -0.58$	$\left(-\frac{\pi}{6}, -\frac{\sqrt{3}}{3}\right)$
0	0	$(0, 0)$
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{3} \approx 0.58$	$\left(\frac{\pi}{6}, \frac{\sqrt{3}}{3}\right)$
$\frac{\pi}{4}$	1	$\left(\frac{\pi}{4}, 1\right)$
$\frac{\pi}{3}$	$\sqrt{3} \approx 1.73$	$\left(\frac{\pi}{3}, \sqrt{3}\right)$



The End

Thanks